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RESEARCH ARTICLE

Benford's Law: Textbook Exercises and Multiple-Cho Testbanks

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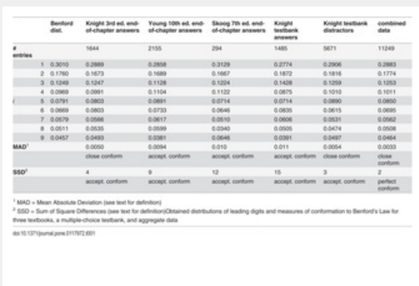
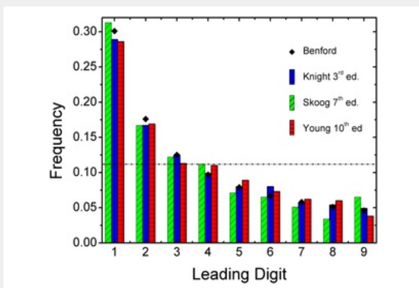
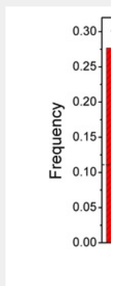
Published: February 17, 2015 • https://doi.org/10.1371/journal.pone.0117972

Figures







Abstract

Benford's Law describes the finding that the distribution of leading (
innumerable datasets follows a well-defined logarithmic trend, rath
uniformity. In practice this means that the most common leading dig
expected frequency of 30.1%, and the least common is 9, with an ex
4.6%. Currently, the most common application of Benford's Law is in
invention and tampering such as found in accounting-, tax-, and vot
demonstrate that answers to end-of-chapter exercises in physics a
textbooks conform to Benford's Law. Subsequently, we investigate v
be used to gain advantage over random guessing in multiple-choic
while testbank answers in introductory physics closely conform to B
testbank is nonetheless secure against such a Benford's attack for

Citation: Slepkov AD, Ironside KB, DiBattista D (2015) Benford's L
Exercises and Multiple-Choice Testbanks. PLoS ONE 10(2): e011
https://doi.org/10.1371/journal.pone.0117972

Academic Editor: William I. Newman, UCLA, UNITED STATES

Received: March 9, 2014; **Accepted:** January 6, 2015; **Published:**

Data Availability: Access to the full data set from the three textb
bank can be obtained by request from Dr. Aaron D. Slepkov at
aaronslepkov@trentu.ca.

Funding: This work was funded in part by an NSERC Discovery G
funders no role in study design, data collection and analysis, de
preparation of the manuscript.

Competing interests: The authors have declared that no compe

Introduction

The expectation that the leading digits in the numbers one encount
uniformly distributed is a well-known "pitfall of elementary statistics
mathematician Simon Newcomb observed that the earlier pages in
tables were considerably more worn than the later pages in the bo
posit that the leading digits of various numbers he encountered we
logarithmically. Fifty years later, physicist Frank Benford independe
same effect in his log tables and came to the same conclusion. Ben

collect over 20,000 numbers from 20 different datasets such as fro
entries, street addresses of eminent scientists, physical constants,
baseball statistics, to show that this finding applies to a wide range
Benford's proposed distribution of leading digit frequencies is giver

where PiB is the probability of finding i as the leading digit in a giver
the frequency of leading digits diminishes monotonically from 30.1%
digits 1 and 2, to 5.1% and 4.6% for the digits 8 and 9, respectively.
finding a "law of anomalous numbers" [3]. Nonetheless, in accordar
eponymy [4], this has come to be known as Benford's Law of leadin

Must there be a well-defined distribution of leading digits for a giver
dataset? Intuitively, most people suspect that the most likely distrib
should be a uniform distribution, such that they are all equally likely
physical phenomenon, any such stable distribution should be scale
the choice of unit system or choice of base would maintain the exis
distribution. For example, if there is to be a well-defined distribution
tabulation of the mass of various insect species, it shouldn't matter
measured in milligrams or in grains of rice. It has been shown that E
only scale invariant distribution of leading digits [5]. Furthermore, su
assures base invariance as well [6]. Thus, it could be said that if a g
an identifiable leading-digit distribution, then it must follow Benford'
there is no requirement that all datasets must have a base-invariar
leading digits, and thus not all phenomena follow Benford's Law.

Decades of research have led to some guidelines for predicting wh
follow Benford's Law [8–10]. Such phenomena should span several
and preferably should be unbounded. The numbers in the dataset s
relevant and should represent a measurement that has associatec
numbers, lottery numbers, and license plate numbers are not expe
distributed. Finally, it has been suggested that the dataset should c
quantities than large quantities [8, 11]. Despite such guidelines, the
examples of sets of numbers that violate at least one of these guide
Benford's law, such as the Fibonacci Numbers [12], and most geome
are now numerous published examples of mathematical series anc
follow Benford's Law [13]. These include geological streamflow rate
lives [15], hadron widths [16], numerous statistical distributions [17,
[19], and business invoices and tax returns [8, 9, 20], as well as the
studied Benford [3]. The precision with which various types of data
has led to its widespread application for forensic accounting and a
while this application represents by far the most active (and comme
there are nonetheless serious practical concerns regarding the va
of this practice [9, 23]. Benford's Law has not yet found robust applic
although from early on there have been suggestions that it could b
applications in computer design and scientific calculation errors [5,
often been a sense that Benford's Law is purely mathematical, rathe

will never find practical applications [24]. Others argue that the law
thus has tremendous potential for future applications [9].

Because only certain datasets that correspond to real phenomena
Benford's Law, we trust that knowledge of this fact could be applied
predictive ends—as opposed to *a posteriori* diagnostics. In consider
guidelines for datasets that are expected to conform to Benford's L
of answers to end-of-chapter questions in introductory physics and
as a possible Benford's Law candidate. In fact, prior research has d
abundance of small-value numbers in grade school math textbooks
arithmetic and multiplication, and has suggested a possible link to
In this article, we test this hypothesis, and find the set of introducto
answers to conform well to the logarithmic distribution of leading di

Confirmation that quantitative answers to a wide range of "problem
chemistry follow Benford's Law immediately suggests a practical (if
application: examination questions that are meant to assess knowl
physics and chemistry should be similar to those found in the pract
textbooks. Thus, multiple-choice testbanks that are created to prov
tool might also follow similar leading-digit trends found in the textbo
true, then perhaps the implication that over 60% of the answers to
choice exam questions are anticipated to have a leading digit of 1,
to gain an advantage by physics-ignorant but test-wise students. T
ignorant student can resort to complete guessing on a multiple-cho
question has an expected baseline for guessing that is based on th
the question. A student has a 33% chance of getting any given ques
question, a 25% chance in a 4-option question, and a 20% chance i
Students might employ a slew of test-wise strategies to boost this b
strategy—which is often derided by professors but may have some
[28]—involves preferential selection of middle options. This manifes
avoidance is colloquially known as "when in doubt, choose C!". This
most others) is based on poor test construction, and is easily foiled
Benford's Law-based attack on a multiple-choice testbank would be
assumption that it is more likely that the incorrect options (distracto
selected in such a way that each yields a uniform first-digit distribut
demonstrating that answers to end-of-chapter textbook questions
distribution, we discuss considerations for which a Benford's Law-b
multiple choice testbank is expected to gain an advantage over ran
then proceed to analyze the distribution of leading digits in an actu
multiple-choice testbank, and go on to find that when we attack the
ubiquity of the Benford distribution in fact secures the bank against

Finally, we discuss how the rounding off of numbers to a pedagogic
reduced set of significant figures is expected to alter the distributio
away from Benford's Law and demonstrate that a modified distribut
yields even better fits to testbank data in physics.

Methods and Results

Data Collection

To select an appropriate representative sample of physics and che
books were chosen based on ready availability on our bookshelf a
popularity in undergraduate physics education:

- "Physics for Scientists and Engineers: A Strategic Approach",
(Pearson Education, 2013) is a highly popular introductory ph
approach that is based on recent physics education researc
- "Sears and Zemansky's University Physics", 10th edition, by Y
(Addison Wesley Longman, 2000) was a popular calculus-ba
physics textbook a decade ago, with emphasis on physics e
fundamentals.
- "Fundamentals of Analytical Chemistry", 7th edition, by Skoog
(Saunders College Publishing, 1996) was a favorite intermed
undergraduate chemistry textbook for one of us (ADS), and v
anticipation of useful quantitative chemistry end-of-chapter

Henceforth these three books will be referred to as Knig
Freedman, and Skoog, respectively. The data was obtai
the leading digit (i.e. the leftmost nonzero digit) from ev
answer. Data collection was implemented manually by p
texts, but with a protocol designed to eliminate subjecti
Nonetheless, because of the general constraints of anti
data sets, such as avoiding unphysical numbers and num
narrowly confined in domain, we rejected all unitless valu
as percentages, and those with units of degrees. Furthe
number zero is meaningless from a significant digit stan
of exactly zero were rejected. Obviously, non-numeric e
answers such as pictures, graphs, equations, and textua
ignored. In all, approximately 10%–15% of entries in the
physics texts were rejected and 30%–35% were ignored
Skoog—the analytical chemistry text—as many as 25%
answers were rejected, largely due to being unitless or p
than these limitations, all other data was recorded. For t
recorded leading digits of the multiple-choice numerical
Knight. When parsing this testbank we looked at both th
and at the distractors, recording the leading digit of each
recording testbank entries, the same protocol was follo
for textbook data; however, the number of rejected entr
was only 7%. In sum, this data is presented in Table 1. T
is freely available as S1 Dataset.

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Table 1. Obtained frequencies of leading digits in textbooks and

https://doi.org/10.1371/journal.pone.0117972.t001

Data Analysis

The data presented in Table 1 includes the theoretically-expected l distributions of Benford's Law. According to Eqn. 1 this distribution c realized in any dataset because the values are irrational numbers. can only be *approached*, and any dataset—no matter how good—w from the ideal distribution. In many cases, where the invocation of B simply to highlight the disproportionate abundance of low-value lea value leading digits, the suggestion of Benford's Law can be confirr inspection of a digit frequency histogram. Such a histogram is pres the distributions for end-of-chapter exercise answers from the thre textbooks clearly yield a Benford-like distribution with an observed frequency with increasing leading digit value. Establishing a bench measure of conformity to Benford's Law has been an ongoing resea 19, 29]. In the Benford's Law literature within the natural sciences, th measure of statistical conformity to a Benford distribution is the $\blacksquare^2$ fit. However, leading expertise in Benford's Law analysis finds that t misused and misinterpreted [9]. The $\blacksquare^2$ test has been described as "excess power" problem, wherein larger data sets require increasi the $\blacksquare^2$ threshold for conformity [8]. Thus, larger data sets that by in fits than smaller datasets will often fail a $\blacksquare^2$ test that the smaller da

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Fig 1. The distribution of leading digits in end-of-chapter excerc two popular introductory physics textbooks (Knight, Young & F analytical chemistry textbook (Skoog).

The dashed horizontal line indicates uniform distribution of first black squares are the theoretical Benford's Law distribution.

https://doi.org/10.1371/journal.pone.0117972.g001

As a way to avoid problems with the ■ 2 test, other simple tests hav
as a means of judging dataset conformity to Benford's Law. For exa
proposed using a mean absolute deviation (MAD) measure [8], and
proposed using a sum of squares difference (SSD) measure [9]. MA
based whole-test measure that simply takes the average of the abs
each digit's frequency from the ideal Benford's Law frequency. Spec

where K is the number of leading digit bins (9 for first leading digit; 9
digits, etc.), AP is the actual proportion observed, and EP is the expe
according to Benford's Law. The MAD test does not have an analytic
value. Instead, Nigrini has established empirically-based criteria for
Benford's Law. [8] The suggested MAD ranges for "close conformity
conformity", and "marginal conformity" are 0.000–0.006, 0.006–0.0
respectively. A MAD value above 0.015 is considered non-conformi

Likewise, SSD too is an empirically-based whole-test measure that s
the squares of the deviation of each digit's frequency from the idea
frequency. Specifically, this is given by

Like the MAD test, the SSD test does not have an analytically-derive
Kossovsky's suggested SSD conformity criteria for "perfect conform
conformity", and "marginal conformity" are 0–2, 2–25, and 25–100, r
value above 100 is considered non-conforming [9].

Results

The three textbooks varied in the number of available entries for ar
contained a sufficient number of entries and domain of data to war
analysis. Skoog provided the smallest data set with a total of 294 en
with several chapters having no usable entries. In total, however, th
this book spanned 60 orders of magnitude. Knight and Young both
sets, with 1644 and 2155 entries in 42 and 39 chapters, respectivel
to physics questions in these books spanned over 170 orders of ma
Table 1, Knight closely conforms to Benford's law (MAD = 0.0050; SS
Freedman (MAD = 0.0094; SSD = 9) and Skoog (MAD = 0.010; SSD = 1
acceptable conformity. A visual inspection of the distribution histog
also strongly suggests close conformity. Thus we conclude that, in g
of-chapter questions in physics and chemistry textbooks follow Ben

The first-digit frequency distribution for the companion multiple-cho
is presented in Fig. 2. The keyed-responses (i.e. correct answers) co
Benford's Law, yielding a MAD value of 0.011 and an SSD value of 1

the answers to numerical multiple-choice exam questions are antic
leading digit of 1, 2, or 3. As mentioned above, a Benford's Law-base
choice testbank would be guided by the assumption that it is more
options (distractors) are semi-randomly selected in such a way that
first-digit distribution. Then the question remains whether the laten
the keyed response will provide more low-digit options than an ens
distributed distractors. The frequency of each *lowest*-leading-digit, *i*
distractors can be given by

where Nk are the standard binomial coefficients (often read as "*N* c
probabilities are presented in Fig. 3, for each of a 3-, 4-, and 5- optio
overlaid with the Benford-distributed correct response.

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**Fig 2. The distribution of leading digits in multiple-choice testba
associated distractors for Knight, "Physics for Scientists and Eng
edition.**

The dashed horizontal line indicates uniform distribution of first
analysis confirms conformation to Benford's Law, overlaid as bla
Conformation of the distractors to Benford's Law precludes a Be
on the testbank.

https://doi.org/10.1371/journal.pone.0117972.g002

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**Fig 3. The frequency of lowest-leading-digits among an ensembl
distributed-fist-digit distractors *N* for 3-, 4-, and 5-option multi
questions, and the expectation of success using a Benford's atta**

For a Benford's Law-based attack on a testbank the predominan
leading digits in the answers must be maintained in the presenc

distracotrs. Despite the fact that for 4- and 5-option questions th
collectively more likely to have the lowest leading digit, a Benfor
group is nonetheless expected to yield an advantage over a ran
strategy (inset). In the case of a test with 3-option questions—wh
are Benford distributed and the two distractors are uniformly dis
Benford attack is expected to yield a passing score of 53% (inse

https://doi.org/10.1371/journal.pone.0117972.g003

From Fig. 3, we see that, while for a 3-option test the lowest leading
in the Benford-distributed keyed responses than in the combined p
at least one distractor in each 4- and 5-option item is expected to h
first digit than the keyed response. These considerations would str
advantage of a Benford's Law-based attack on a set of multiple-cho
would not entirely negate this strategy. This is demonstrated by co
probability of finding the correct answer by choosing the lowest lea
group of N uniformly-distributed-first-digit-distractors and a Benfor
answer, with random guessing in cases where the lowest leading d
given by

where PiB are the Benford probabilities as given by Eq. 1. This expre
that presented in a recent note by F. M. Hoppe [31], and is plotted fo
choice questions in the inset to Fig. 3. Using Eq. 5 we find that such
improves the test score over random guessing: For a 3-, 4-, or 5-opt
scores of 53%, 44%, or 38%, respectively. Compared to blind-guess
and 20%, a Benford attack promises a significant advantage. The e
passing score in a 3-option exam is particularly noteworthy conside
recommendations that this is (psychometrically) the most desirable
item [32].

We attacked the Knight testbank by selecting the option with the lo
guessing among items with identical lowest leading digits, and com
to the keyed response. The majority of questions were of the 4-opti
yielded a score of 24.6%; i.e. no better than chance. The explanatio
strategy lies in the first-digit distribution of the distractors. As show
Table 1), the testbank distractors also closely conform to Benford's
strictest MAD criteria. Thus, the fact that the keyed responses are B
marginalized by the likewise distributed distractors.

Discussion

As we have shown, typical physics and chemistry questions, as a g
Law for leading digits, as do both the keyed responses and the dist
introductory physics testbank. Uniformly-distributed random numbe
follow Benford's Law, but rather have uniformly-distributed leading

definite conclusion we can draw from our analysis of the testbank i
multiple-choice questions the distractors are clearly not random nu
interesting question is whether the fact that the distractors follow th
leading digits as do the keyed-responses—which emerge from wel
deterministic procedures—means that they too are a result of a sim
That is, are the distractors necessarily created as answers to altern
necessarily. While *de novo* random numbers are uniformly distribute
processed random numbers are not, and multiplications of arrays o
known to generate near-ideal Benford sets [33]. Furthermore, a ran
numbers from multiple sets of different distributions—none of whic
distributed—yields a Benford set. This appears to be a leading-digit
central limit theorem [3, 6]. Finally, the scale invariance of Benford's
multiplying values within a Benford set by various other numbers m
distribution of first digits. Thus, we can not identify the way in which
created. Whether they are made of processed random numbers, pr
keyed response, or are themselves erroneous but plausible answe
cannot be discerned from their distribution of leading digits; any of
Benford Distribution.

The question of why a large but seemingly random subset of all pos
physics questions should follow Benford's Law with such precision i
may not be a clear-cut answer to this question, but the truth probab
aforementioned theorem that random sampling from a wide mixture
distribution sets converges to a Benford distribution. As a group, er
questions (or potential final examination questions) span many top
numerous different parameters, each of which may have a differen
within the domain of physically relevant phenomena. As examples,
"kinetic energies for vehicles on earth" and realistic "currents induc
Lenz's Law" may be sufficiently limited in domain as to individually re
distribution, but random sampling of values across such distributio
set. This is likely the reason that such a good Benford distribution is
As further evidence of this argument, we see that combining the da
textbooks, the testbank answers, and the testbank distractors yield
set that better conforms to Benford's Law than any of the individual
Table 1, this combined dataset shows a minuscule MAD of 0.0033 a
"perfect Benford".

Thus far we have determined that answers to physics questions co
Law. However, these numerical quantities, as found in the textbook
are often reported to an artificially reduced number of significant d
rounding and Benford's Law was identified at inception, [3] and has
some potential applications of the law for computer design [5, 34]. T
reported in the Benford's Law literature contain many more than thr
For pedagogical reasons, numbers reported in introductory physics
the smallest number of significant digits warranted by the precision
the calculation or measurement. Instructional material with number
two or three significant figures may thus represent an artificial data
Bendord distribution of leading digits. A description of how rounding
distribution of leading digits is fairly straightforward: In the case of f

the effect of rounding is to diminish the expected number of 1s and
frequency of all other digits. We present a modification to Eq.1 that
rounding, where N_{SD} is the number of significant digits to which a v
N_{SD} = □ represents the traditional case of no rounding:

For numbers rounded to three or more significant digits the distribu
is nearly identical to Benford's Law. However, in the case of values r
significant digits, there is a small but significant modification of the
numbers are rounded to only one significant digit, the distribution o
varies drastically from Benford's Law to the point where the occurre
becomes more probable than the digit 1. Table 2 summarizes this re
that a dataset comprised of values rounded to two significant digits
both the MAD and SSD tests of conformity to Benford's law, but a da
rounded to a single significant digit would fail to conform to a Benfo
the conformity of the underlying phenomena (i.e. the un-rounded da
observed that approximately 30% of entries in the testbank are rep
digits. Thus, perhaps for the testbank a more appropriate distributio
than that given by Eq.1 should be given by a respective 0.7:0.3 weig
Eqns.1 and 6. When compared to this hybrid distribution, we obtain
which is slightly better than the MAD value of 0.0045 of this full testb
unmodified Benford's Law. Likewise, the SSD reduces from 3 to 2, ma
data's fit to the hybrid distribution. Thus, we are likely observing the
our data, and this effect is expected to be a factor in the leading-dig
similar dataset where rounding to one or two significant figures is c

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Table 2. Effects of rounding on Benford distribution.

https://doi.org/10.1371/journal.pone.0117972.t002

Summary and Conclusions

We recorded the leftmost significant digit of the answers to every e
in two popular introductory physics textbooks, an intermediate ana
textbook, and a large introductory physics multiple-choice testbank
conform to Benford's Law. The fact that the answers to multiple-cho
follow this trend suggested a means by which the testbank could be

subject-ignorant but test-wise student. We find that among a set of answers), each having uniformly distributed leading digits, the Benf keyed response could be used to pass a 3-option test. Nonetheless information to "guess" the correct answers in a real testbank we fir themselves conform to Benford's Law, thereby securing the testbar The fact that the distractors are Benford distributed does not, howe were engineered intelligently, but simply suggests that they do not distributed random values. Thus, we expect any such testbank to b against a Benford's Law based attack. We observed that physics te items are often reported rounded to two significant digits, and we h fact may have impacted its distribution of leading digits, modifying i law and primarily yielding a relative dearth of leading 1s. We expec our demonstration of Benford's Law in end-of-chapter textbook ans counter-intuitive, just as math educators were surprised that grade biased toward small addition and multiplication facts [25, 26]: most p that the leading digits of random values, and the numbers in arithm be uniformly distributed. Armed with the knowledge that answers to follow Benford's law, one trifling piece of advice we can give to the t follows: at the end of a long constructed-response examination, if y double-check the answers to all of the questions, spend time on tho yielded final answers that have the largest leading digits; questions answers with leading digits 7, 8, or 9 only 15% of the time. You shou any advantages of using Benfords Law will only pay dividends if wo set, and thus any advantages in test-taking would be, at most, marg

Supporting Information

S1 Dataset. The full raw data and example statistical analysis file.

Worksheet "Data" includes raw data and summary counts/proporti five principle sources described in the article. Worksheet "Stat Ana example of the data analysis preformed on one of the data sources
https://doi.org/10.1371/journal.pone.0117972.s001
(XLS)

Acknowledgments

We thank Fred Hoppe, of McMaster University, for his interest and c pre-print, as well as for his contribution to the derivation of the close presented in Eq. 5. We thank Alex Kossovsky for extensive and fruit revised manuscript. We thank Pearson Education for the introducto choice testbank.

Author Contributions

Conceived and designed the experiments: ADS DD. Performed the e
Analyzed the data: KBI ADS. Contributed reagents/materials/analys
the paper: ADS DD.

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30. Indeed, in our study we find the χ^2 test to suffer from such e
Generally we find that our small data sets such as Knight an
chapter answers pass the χ^2 threshold test (15.5 for p<0.05
freedom), while the larger data sets such as the testbank dis
end-of-chapter answers marginally fail the test despite the f
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