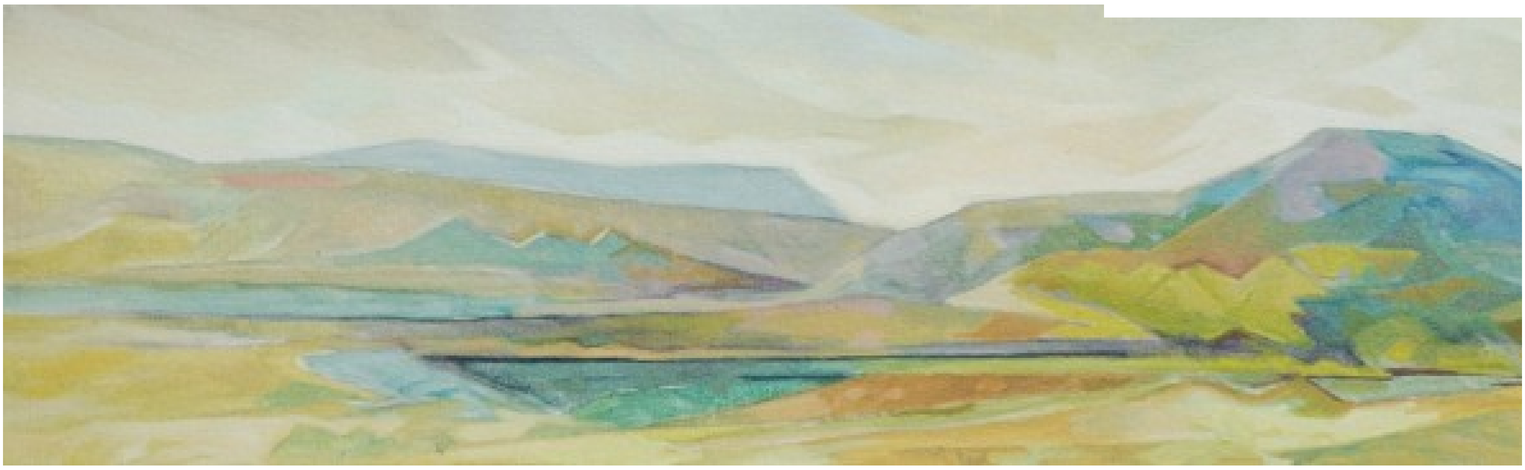

Foundations of Mathematics, Logic & Compu

Reviewing classical interpretations of Cantor's, Gödel's, Tarski's, and Turin and addressing some grey areas in the foundations of mathematics, logic a computability

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The Holy Grail of Arithmetic: Bridg Provability and Computability

November 11, 2013 in Foundations of mathematics, logic and computation, algorithmically computable, algorithmically verifiable, Aristotle's particular Bringsjord, Church Turing Thesis, completed infinity, computationalism, c finitary, first order, FOL, formal, formal language, Fortnow, foundations, Gö Goldin, Hilbert, human, human intelligence, logic, mathematical truth, math intelligence, mechanist, Mendelson, natural numbers, omega-consistency, Peano Arithmetic PA, Peter Wegner, philosophy, reasoning, Rosser, Rule C simple functional language, standard interpretation, syntactic, Tarski, Turin Turing machines, undecidable, unpredictability, Wegner, Wilfrid Sieg

§1 The Holy Grail of Arithmetic: Bridging Provability and Co

See also this update.

(*Notations, non-standard concepts, and definitions used commonly in are detailed in this post.*)

Peter Wegner and Dina Goldin

In a short opinion paper, `Computation Beyond Turing Machines', C Peter Wegner and Dina Goldin (Wg03) advanced the thesis that:

> `A paradigm shift is necessary in our notion of computational can provide a complete model for the services of today's com software agents.'

We note that Wegner and Goldin's arguments, in support of their the extraordinarily eclectic view of mathematics, combining both an imp and implicit frustration at, the standard interpretations and dogmas c

Why Gödel's 'undecidable' formula does not assert its own unprovability

The enduring evolution of logic

Does AI pose an existential threat to mankind?

Can Primes yield a heuristic model of quantum behaviour?

The *PvNP* problem: *Why* we need to review current approaches towards determining the computational complexity of *Integer Factorising*

Can we think about *the infinite* in a *consistent way*, and would such a concept be *helpful*?

The Unexplained Intellect

Bridging the Finite-Infinite Divide?

Should the foundations of mathematics be arithmetical and not set-theoretical?

Three intriguing non-heuristic prime counting function queries

The Evidence-Based Argument for Lucas' Gödelian Thesis

Generating primes sequentially

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A Highly Speculative Post About Speculation: Defining Market Equilibrium in a Simplified Stock Exchange

What differentiates the Finitary Logic of these investigations from Classical Hilbertian Logics and Intuitionistic Brouwerian Logics?

Why we need to define Effective Computability formally and weaken the Church and Turing Theses – II

Why we need to define Effective Computability formally and weaken the Church and Turing Theses – I

Why we shouldn't fault J. R. Lucas and Roger Penrose for their Gödelian arguments against computationalism – II

The case against non-standard models of PA – II

The Holy Grail of Arithmetic: Bridging Provability and Computability

PA is finitarily consistent: A solution to the Second of Hilbert's Twenty Three Problems

mathematical theory:

> (i) `… Turing machines are inappropriate as a universal founda
> problem solving, and … computer science is a fundamentally
> discipline.'

> (ii) `(Turing's) 1936 paper … proved that mathematics could n
> modeled by computers.'

> (iii) `… the Church-Turing Thesis … equated logic, lambda cal
> machines, and algorithmic computing as equivalent mechanis
> solving.'

> (iv) `Turing implied in his 1936 paper that Turing machines …
> model for all forms of mathematics.'

> (v) `… Gödel had shown in 1931 that logic cannot model math
> showed that neither logic nor algorithms can completely mode
> human thought.'

These remarks vividly illustrate the dilemma with which not only The
Sciences, but all applied sciences that depend on mathematics—fo
language to express their observations precisely—are faced:

> **Query:** Are formal classical theories essentially unable to ade
> extent and range of human cognition, or does the problem lie
> theories are classically interpreted at the moment?

The former addresses the question of whether there are absolute li
express human cognition unambiguously; the latter, whether there a
limits—not necessarily absolute—to the capacity of classical interpr
communicate unambiguously that which we intended to capture wit
expression.

Prima facie, applied science continues, perforce, to interpret mathe
Platonically, whilst waiting for mathematics to provide suitable, and
answers as to how best it may faithfully express its observations ve

Lance Fortnow

This dilemma is also reflected in Computer Scientist Lance Fortnow
Wegner and Goldin's thesis, and of their reasoning.

Thus Fortnow divides his faith between the standard interpretations
mathematics (and, possibly, the standard set-theoretical models of
as standard Peano Arithmetic), and the classical computational the
machines.

He relies on the former to provide all the proofs that matter:

> `Not every mathematical statement has a logical proof, but log
> everything we can prove in mathematics, which is really what m

and, on the latter to take care of all essential, non-provable, truth:

> `… what we can compute is what computer science is all abou

Can faith alone suffice?

Misunderstanding Gödel: The significance of Feynman's cover-up factor

Meeting Wittgenstein's requirement of 'truth' in Gödel's formal reasoning

The PvNP Separation Problem

Notation, non-standard concepts and definitions used in these investigations

The reals are denumerable: The finitary solution that Hilbert probably would not have foreseen for the First of his Twenty Three problems!

How mechanical intelligences and human intelligences can reason in contradictory, yet complementary, ways!

Mathematics does not need a new foundation; it only needs to make its implicit assumptions explicit!

Does mathematics need a new foundation?

Beyond Hawking's philosophical pronouncement

Is philosophy really dead?

Do we want our Giants to be our step-ladders or our white canes?

From perfect numbers to a generating function for the unrestricted factorisations of an integer

A suggested mathematical perspective for the EPR argument

Why we shouldn't fault J. R. Lucas and Roger Penrose for their Gödelian arguments against computationalism – I

The Mechanist's Challenge to John R. Lucas

Can someone tell me what is so special about Rosser's proof of formally undecidable arithmetical propositions?

Placing Cohen's proof of the Independence of the Axiom of Choice in perspective

Why Brouwer was justified in his objection to Hilbert's unqualified interpretation of quantification

A foundational perspective on the semantic and logical Paradoxes – IV

A foundational perspective on the

However, as we shall argue in a subsequent post, Fortnow's faith i
Turing Thesis that ensures:

> \`… Turing machines capture everything we can compute',

may be as misplaced as his faith in the infallibility of standard interp
mathematics.

The reason: There are, prima facie, reasonably strong arguments fo
paradigm shift; not, as Wegner and Goldin believe, in the notion of
problem solving, but in the standard interpretations of classical ma

However, Wegner and Goldin could be right in arguing that the dire
must be towards the incorporation of non-algorithmic effective met
mathematical theory (as detailed in the Birmingham paper); presum
remarks, that this is, indeed, what \`external interactions' are assum
classical Turing-computability:

> (vi) \`… that Turing machine models could completely describe
> computation … contradicted Turing's assertion that Turing ma
> formalize algorithmic problem solving … and became a dogm
> theory of computation'.
>
> (vii) \`… interaction between the program and the world (enviro
> during the computation plays a key role that cannot be replace
> determined prior to the computation'.
>
> (viii) \`… a theory of concurrency and interaction requires a new
> framework, not just a refinement of what we find natural for se
> computing'.
>
> (ix) \`… the assumption that all of computation can be algorith
> widely accepted'.

A widespread notion of particular interest, which seems to be recur
Wegner and Goldin's assertions too, is that mathematics is a dispe
science, rather than its indispensable mother tongue.

Elliott Mendelson

However, the roots of such beliefs may also lie in ambiguities, in th
of foundational elements, that allow the introduction of non-constru
verifiable, non-computational, ambiguous, and essentially Platonic
standard interpretations of classical mathematics.

For instance, in a 1990 philosophical reflection, Elliott Mendelson's
Me90; reproduced from Selmer Bringsjord (Br93)), implicitly imply t
definitions of various foundational elements can be argued as bein
non-constructive, or both:

> \`Here is the main conclusion I wish to draw: it is completely un
> CT is unprovable just because it states an equivalence betwee
> notion (effectively computable function) and a precise mathem
> recursive function). … The concepts and assumptions that sup
> partial-recursive function are, in an essential way, no less vag
> the notion of effectively computable function; the former are jus
> are part of a respectable theory with connections to other parts

semantic and logical paradoxes – III
The Butterfly Effect
A foundational perspective on the semantic and logical paradoxes – II
A foundational perspective on the semantic and logical paradoxes – I
The case against Goodstein's Theorem – IX
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The case against Goodstein's Theorem – IV
The case against Goodstein's Theorem – III
The case against Goodstein's Theorem – II
The case against Goodstein's Theorem – I
Which is the canonical model of PA?
The case against non-standard models of PA – I
Is Gödel's undecidable proposition an `ad hoc' anomaly?
Let not posterity judge us as having spent our lives polishing the pebbles and tarnishing the diamonds

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> mathematics. (The notion of effectively computable function co
> incorporated into an axiomatic presentation of classical mathe
> acceptance of CT made this unnecessary.) … Functions are de
> but the concept of set is no clearer than that of function and a fo
> mathematics can be based on a theory using function as primi
> set. Tarski's definition of truth is formulated in set-theoretic ter
> set is no clearer than that of truth. The model-theoretic definitio
> based ultimately on set theory, the foundations of which are no
> intuitive understanding of logical validity. … The notion of Turin
> function is no clearer than, nor more mathematically useful (fo
> than, the notion of an effectively computable function.'

Consequently, standard interpretations of classical theory may, inac
weakening a desirable perception—of mathematics as the lingua fra
expression—by ignoring the possibility that, since mathematics is, i
accepted as the language that most effectively expresses and comm
truth, the chasm between formal truth and provability must, of neces

Cristian Calude, Elena Calude and Solomon Marcus

The belief in the existence of such a bridge is occasionally implicit in
computational theory.

For instance, in an arXived paper *Passages of Proof*, Computer Scie
Elena Calude and Solomon Marcus remark that:

> "Classically, there are two equivalent ways to look at the math
> proof: logical, as a finite sequence of sentences strictly obeyin
> inference rules, and computational, as a specific type of comp
> proof given as a sequence of sentences one can easily constr
> producing that sequence as the result of some finite computati
> given a machine computing a proof we can just print all senten
> the computation and arrange them into a sequence."

In other words, the authors seem to hold that Turing-computability o
of an arithmetical proposition, is equivalent to provability of its repre

Wilfrid Sieg

We now attempt to build such a bridge formally, which is essentially
arithmetical 'Decidability and Calculability' described by Philosophe
depth and wide-ranging survey 'On Comptability', in which he addre
belief that an iff bridge between the two concepts is 'impossible' for
predicates' (Wi08, p.602).

§2 Bridging provability and computability: The foundations

In the paper titled "Evidence-Based Interpretations of PA" that was
Symposium on Computational Philosophy at the AISB/IACAP World
Turing 2012, held from 2^{nd} to 6^{th} July 2012 at the University of Birm
(reproduced in this post) we have defined what it means for a numbe
be:

(i) Algorithmically verifiable;

(ii) Algorithmically computable.

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Shtetl-Optimized
Nanoexplanations
Eric Cavalcanti
East Asia Forum
Azimuth
Tanya Khovanova's Math Blog
The polymath blog

We have shown there that:

(i) The standard interpretation $\mathcal{I}_{PA(N,\ Standard)}$ of the first ord
PA is finitarily sound if, and only if, Aristotle's particularisation
latter is the case if, and only if, PA is ω-consistent.

(ii) We can define a finitarily sound algorithmic interpretation $\mathcal{I}$
PA over the domain N where, if $[A]$ is an atomic formula $[A(x$
then the sequence of natural numbers $(a_1, a_2, \ldots, a_n)$ satisfie
$[A(a_1, a_2, \ldots, a_n)]$ is algorithmically computable under $\mathcal{I}_{PA(N}$
do not presume that Aristotle's particularisation is valid over N

(iii) The axioms of PA are always true under the finitary interpre
$\mathcal{I}_{PA(N,\ Algorithmic)}$, and the rules of inference of PA preserve
satisfaction/truth under $\mathcal{I}_{PA(N,\ Algorithmic)}$.

We concluded that:

Theorem 1: The interpretation $\mathcal{I}_{PA(N,\ Algorithmic)}$ of PA is fi

Theorem 2: PA is consistent.

§3 Extending Buss' Bounded Arithmetic

One of the more significant consequences of the Birmingham paper
the iff bridge between the domain of provability and that of computab
Buss' Bounded Arithmetic by showing that an arithmetical formula $[$
and only if, $[F]$ interprets as true under an algorithmic interpretation

§4 A Provability Theorem for PA

We first show that PA can have no non-standard model (for a distin
this convention-challenging thesis see this post and this paper), si
`algorithmically' complete in the sense that:

Theorem 3: (*Provability Theorem for PA*) A PA formula $[F(x)]$
only if, $[F(x)]$ is algorithmically computable as always true in

Proof: We have by definition that $[(\forall x)F(x)]$ interprets as true
interpretation $\mathcal{I}_{PA(N,\ Algorithmic)}$ if, and only if, $[F(x)]$ is algo
as always true in N.

Since $\mathcal{I}_{PA(N,\ Algorithmic)}$ is finitarily sound, it defines a finitar
—say $\mathcal{M}_{PA(\beta)}$—such that:

If $[(\forall x)F(x)]$ is PA-provable, then $[F(x)]$ is algorithmical
always true in N;

If $[\neg(\forall x)F(x)]$ is PA-provable, then it is not the case that
algorithmically computable as always true in N.

Now, we cannot have that both $[(\forall x)F(x)]$ and $[\neg(\forall x)F(x)]$ a
some PA formula $[F(x)]$, as this would yield the contradiction:

(i) There is a finitary model—say $M1_\beta$—of PA+$[(\forall x)F(x$
algorithmically computable as always true in N.

(ii) There is a finitary model—say $M2_{\beta}$—of PA+$[\neg(\forall x)F$
the case that $[F(x)]$ is algorithmically computable as alw

The lemma follows. □

§5 The holy grail of arithmetic

We thus have that:

Corollary 1: PA is categorical finitarily.

Now we note that:

Lemma 2: If PA has a sound interpretation $\mathcal{I}_{PA(N,\ Sound)}$ ov
PA formula $[F]$ which is algorithmically verifiable as always tru
$\mathcal{I}_{PA(N,\ Sound)}$ even though $[F]$ is not PA-provable.

Proof In his seminal 1931 paper on formally undecidable arit
Kurt Gödel has shown how to construct an arithmetical formul
—say $[R(x)]$ [1]—such that $[R(x)]$ is not PA-provable [2], but $[R$
PA-provable for any given PA numeral $[n]$. Hence, for any give
formula $xB\lceil[R(n)]\rceil$ must hold for some x. The lemma follow

By the argument in Theorem 3 it follows that:

Corollary 2: The PA formula $[\neg(\forall x)R(x)]$ defined in Lemma

Corollary 3: Under any sound interpretation of PA, Gödel's $[$
algorithmically verifiable, but not algorithmically computable, t

Proof Gödel has shown that $[R(x)]$ [3] interprets as an algorit
tautology [4]. By Corollary 2 $[R(x)]$ is not algorithmically compu
in N. □

Corollary 4: PA is *not* ω-consistent. [5]

Proof Gödel has shown that if PA is consistent, then $[R(n)]$ is
given PA numeral $[n]$ [6]. By Corollary 2 and the definition of ω-
consistent then it is *not* ω-consistent. □

Corollary 5: The standard interpretation $\mathcal{I}_{PA(N,\ Standard)}$ of
sound, and does not yield a finitary model of PA [7].

Proof If PA is consistent but not ω-consistent, then Aristotle's
not hold over N. Since the `standard', interpretation of PA app
particularisation, the lemma follows. □

Since formal quantification is currently interpreted in classical logic [
Aristotle's particularisation over N as axiomatic [9], the above sugg
to review number-theoretic arguments [10] that appeal unrestrictedly
Aristotlean logic.

§6 The Provability Theorem for PA and Bounded Arithmeti

In a 1997 paper [11], Samuel R. Buss considered Bounded Arithmet

(a) limiting the applicability of the Induction Axiom Schema in P
with quantifiers bounded by an unspecified natural number bou

(b) `weakening' the statement of the axiom with the aim of diffe
effective computability over the sequence of natural numbers,
`polynomial-time' computability over a bounded sequence of t
[12].

Presumably Buss' intent—as expressed below—is to build an iff bri
provability in a Bounded Arithmetic and Computability so that a Π_k f
, is provable in the Bounded Arithmetic if, and only if, there is an algo
given numeral $[n]$, decides the $\Delta_{(k/(k-1))}$ formula $[f(n)]$ as `true':

If $[(\forall x)(\exists y)f(x, y)]$ is provable, then there should be an algori
function of x [13].

Since we have proven such a Provability Theorem for PA in the prev
question arises:

§7 Does the introduction of bounded quantifiers yield any advantage?

Now, one difference [14] between a Bounded Arithmetic and PA is th
the Bounded Arithmetic that, from a proof of $[(\exists y)f(n, y)]$, we may a
there is some numeral $[m]$ such that $[f(n, m)]$ is provable in the arit
is not a finitarily sound conclusion in PA.

Reason: Since $[(\exists y)f(n, y)]$ is simply a shorthand for $[\neg(\forall y)\neg f(n,$
presumption implies that Aristotle's particularisation holds over the
under any finitarily sound interpretation of PA.

To see that (as Brouwer steadfastly held) this may not always be the
$[(\forall x)f(x)]$ as [15]:

There is an algorithm that decides $[f(n)]$ as `true' for any given

In such case, if $[(\forall x)(\exists y)f(x, y)]$ is provable in PA, then we can onl

There is an algorithm that, for any given numeral $[n]$, decides t
that there is an algorithm that, for any given numeral $[m]$, decid
`true'.

We cannot, however, conclude—as we can in a Bounded Arithmetic

There is an algorithm that, for any given numeral $[n]$, decides t
algorithm that, for some numeral $[m]$, decides $[f(n, m)]$ as `tru

Reason: $[(\exists y)f(n, y)]$ may be a Halting-type formula for some num

This could be the case if $[(\forall x)(\exists y)f(x, y)]$ were PA-*un*provable, bu
provable for any given numeral $[n]$.

Presumably it is the belief that any finitarily sound interpretation of P
particularisation to hold in N, and the recognition that the latter does
provability to computability in PA, which has led to considering the e
quantification in PA.

However, as we have seen in the preceding sections, we are able to
computability through the Provability Theorem for PA by recognising
contrary, any interpretation of PA which requires Aristotle's particula
cannot be finitarily sound!

The postulation of an unspecified bound in a Bounded Arithmetic in
provability-computability link thus appears dispensible.

The question then arises:

§8 Does `weakening' the PA Induction Axiom Schema yield
advantage?

Now, Buss considers a bounded arithmetic S_2 which is, essentially
`weakened' Induction Axiom Schema, PIND [16]:

$$[\{f(0)\ \&\ (\forall x)(f(\lfloor \frac{x}{2} \rfloor) \rightarrow f(x))\} \rightarrow (\forall x)f(x)]$$

However, PIND can be expressed in first-order Peano Arithmetic PA

$$[\{f(0)\ \&\ (\forall x)(f(x) \rightarrow (f(2*x)\ \&\ f(2*x+1)))\} \rightarrow (\forall x)$$

Moreover, the above is a particular case of PIND(k):

$$[\{f(0)\ \&\ (\forall x)(f(x) \rightarrow (f(k*x)\ \&\ f(k*x+1)\ \&\ \ldots\ \&$$
$$\rightarrow (\forall x)f(x)].$$

Now we have the PA theorem:

$$[(\forall x)f(x) \rightarrow \{f(0)\ \&\ (\forall x)(f(x) \rightarrow f(x+1))\}]$$

It follows that the following is also a PA theorem:

$$[\{f(0)\ \&\ (\forall x)(f(x) \rightarrow f(x+1))\} \rightarrow$$
$$\{f(0)\ \&\ (\forall x)(f(x) \rightarrow (f(k*x)\ \&\ f(k*x+1)\ \&\ \ldots\ \&\ f$$

In other words, for any numeral $[k]$, PIND(k) is equivalent in PA to th
Axiom of PA!

Thus, the Provability Theorem for PA suggests that all arguments ar
Bounded Arithmetic can be reflected in PA without any loss of gener

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Notes

Return to 1: Gödel refers to this formula only by its Gödel number r

Return to 2: Gödel's immediate aim in Go31 was to show that $[(\forall x)$
provable; by Generalisation it follows, however, that $[R(x)]$ is also r

Return to 3: Gödel refers to this formula only by its Gödel number r

Return to 4: Go31, p.26(2): "$(n)\neg(nB_{\kappa}(17Gen\ r))$ holds".

Return to 5: This conclusion is contrary to accepted dogma. See, fo
remarks in Da82, p.129(iii) that:

"… there is no equivocation. Either an adequate arithmetical logic is
which case it is possible to prove false statements within it) or it has
decision problem and is subject to the limitations of Gödel's incom

Return to 6: Go31, p.26(2).

Return to 7: I note that finitists of all hues—ranging from Brouwer Br
Yessenin-Volpin He04—have persistently questioned the finitary so
`standard' interpretation $\mathcal{I}_{PA(N,\ Standard)}$.

Return to 8: See Hi25, p.382; HA28, p.48; Be59, pp.178 \& 218.

Return to 9: In the sense of being intuitively obvious. See, for instan
Rg87, p.308 (1)-(4); EC89, p.174 (4); BBJ03, p.102.

Return to 10: For instance Rosser's construction of an undecidable
proposition in PA (see Ro36)—which does not explicitly assume th
—implicitly presumes that Aristotle's particularisation holds over N

Return to 11: Bu97.

Return to 12: See also Pa71.

Return to 13: See Bu97.

Return to 14: We suspect the only one.

Return to 15: We have seen in the earlier sections that such an inter
sound.

Return to 16: Where $\lfloor \frac{x}{2} \rfloor$ denotes the largest natural number lower b

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